

1 Properties of Integral

1.

$$\int_a^b f(x)dx = - \int_b^a f(x)dx, \quad \int_a^a f(x)dx = 0$$

2.

$$\int_a^b \text{CONSTANT } dx = \text{CONSTANT} \cdot (b - a)$$

3.

$$\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx, \quad \int_a^b cf(x)dx = c \int_a^b f(x)dx$$

4.

$$\int_a^b [c_1f(x) + c_2g(x)]dx = c_1 \int_a^b f(x)dx + c_2 \int_a^b g(x)dx$$

5.

$$\boxed{\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx}$$

6.

- If $f(x) \geq 0$, then $\int_a^b f(x)dx \geq 0$.
- If $f(x) \geq g(x)$, then $\int_a^b f(x)dx \geq \int_a^b g(x)dx$.
- If $m \leq f(x) \leq M$, then $m(b - a) \leq \int_a^b f(x)dx \leq M(b - a)$

2 Fundamental Theorem of Calculus (FTC)

1. Differential Rule:

$$\text{If } g(x) = \int_a^x f(t)dt, \quad \text{then } g'(x) = \frac{d}{dx}g(x) = f(x)$$

$$\boxed{\text{If } g(x) = \int_{v(x)}^{u(x)} f(t)dt, \quad \text{then } g'(x) = \frac{d}{dx}g(x) = f[u(x)] \cdot u'(x) - f[v(x)] \cdot v'(x)}$$

2.

$$\int_a^b f(x)dx = F(x)|_a^b = F(b) - F(a), \quad \text{where } F'(x) = f(x)$$